ASPECTS REGARDING THE OPTIMIZATION OF THE QUALITY OF MANAGERIAL DECISIONS - SOLVING AN OPTIMIZATION PROBLEM IN THE SENSE OF SUSTAINABILITY WITH WINQSB

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Abstract:

In this paper we present an optimization problem solving, problem that is mathematically shaped as a linear programming problem. This is made with the purpose of finding an optimal solution which is also a sustainable one. In the present and future society, it is clear that nothing can be done honestly, but only in the sense of a professional and sustainable thinking (Konrad, C., 2011, pp.202-217, Petrescu, I., 2006, p.1). To think sustainable can also mean "to be calculated", in popular language "to choose wisely", all for achieving the proposed project, being careful to using non-renewable resources, to pollution, to protection for human resource, etc. Of course that the logical and precise language of Mathematics can help us formulate these issues as rigorous as possible. The problem addressed in this article was simulated in WinQSB software.

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2000 Mathematics Subject Classification: $91B_{30}$, $91B_{99}$, $91D_{99}$

1. Introduction

In an organization, resources, raw materials, machinery, energy, employment, money, etc., are limited. The more so if we take into account sustainable development policies. As a result, managers often have to face decisions about how they should allocate scarce resources in a way that optimizes certain objectives. *Linear programming* is a powerful tool that can help managers to solve a problem of allocation.

In linear programming problems, there come some decisional variables, a set of constraints (restrictions) and the objective function.

The decisional variables in the case of linear programming depend on the type of problem. These can be quantities of resources that must be allocated or a number of units that must be achieved. The decisional variables represent factors that the decider may control. The decider should search values for these variables (usually noted $x_1, x_2,...$ or $x,y,z,...$) so that the optimal solution for that problem to be achieved.

The objective function is a symbolical and quantitative representation of the purpose that the decider tries to optimize. The decider can follow, depending on the problem, either maximize (such as, for ex., for profit) or minimize (such as, for example, in the case of cost or time).

Constraints arise because the decider, in his searching of the decisional variables process that would maximize or minimize the value of the objective function, must comply with certain rules and regulations. These are algebraic statements, represented by some equations or inequalities, reflecting the restrictions on the flexibility of the decider in choosing the right decision (Gorski, H., 2008, pp.70-111).

Linear programming problems can be simulated using WinQSB, LINGO software, etc (Roşca, L., 2008, pp. 66-74).

2. Case study. Mathematical model

The case study refers to achieving in the pharmaceutical industry of a production plan that maximizes the profit, but at the same time – using the conditions imposed by a sustainable development of industries – produces a direction that has to be taken into regard in all present or future projects (Ekins, P., 2000, pp.12-50; Mc Donagh, P., 1997, pp.5-13).

A pharmaceutical industry produces four varieties of medicines, named M_1, M_2, M_3, M_4 for kidney treatment. Manufacture of medicines brings profit, but produces toxic waste T_1, T_2, T_3 depending on the amount of drugs produced:

- to produce 100kg of medicine M_1, occur 20kg of toxic waste T_1,10kg of toxic waste T_2, 20kg of toxic waste T_3;

- to produce 100kg of medicine M_2, occur 20kg of toxic waste T_1,30kg of toxic waste T_2, 40kg of toxic waste T_3;

- to produce 100kg of medicine M_3, occur 10kg of toxic waste T_1,20kg of toxic waste T_2, 30kg of toxic waste T_3;

- to produce 100kg of medicine M_4, occur 40kg of toxic waste T_1,0kg of toxic waste T_2, 10kg of toxic waste T_3.

The profit gained by selling product M_1 is 15 m.u., by selling product M_2, 60 m.u., by selling product M_3 of 4m. u. and by selling medicine M_4 is of 20 m.u.

Current legislation limits the amount of toxic waste T_1, T_2, T_3 at, respectively, 21kg, 6kg and 14kg.

Let us calculate respecting the laws regarding toxic waste the amount produced for each type of medicine that maximizes the profit.

The mathematical model associated to the problem.

Denote by:

x_1- amount of medicine M_1;

x_2- amount of medicine M_2;

x_3- amount of medicine M_3;

x_4- amount of medicine M_4.

The objective function, that we want to maximize and that represents the profit, the benefit, brought by selling the produced medicines, has the following analytical expression:

$$f(x_1, x_2, x_3, x_4) = 15x_1 + 60x_2 + 4x_3 + 20x_4, \qquad (1)$$

where,

- $15x_1$ is the profit brought by selling the quantity x_1 of medicine M_1, taking into account that the profit obtained by selling one unit is 15 m.u.;

- $60x_2$ is the profit brought by selling the quantity x_2 of medicine M_2, taking into account that the profit obtained by selling one unit is 60 m.u.;

- $4x_3$ is the profit brought by selling the quantity x_3 of medicine M_3, taking into account that the profit obtained by selling one unit is 4 m.u.;

- $20x_4$ is the profit brought by selling the quantity x_4 of medicine M_4, taking into account that the profit obtained by selling one unit is 20 m.u..

To produce 100kg of medicine M_1, occur 20kg of toxic waste T_1, so for one unit from M_1 occur 0,2kg of toxic waste T_1, and for x_1 units from M_1 occur $0,2x_1$ kg of toxic waste T_1. Similarly, in the case of M_2. Also, for producing 100kg of medicine M_3, occur 10kg of toxic waste T_1, so for one unit from M_3 occur 0,1kg of toxic waste T_1, and for x_3 units from M_3 occur $0,1x_3$ kg of toxic waste T_1; and, for producing 100kg of medicine M_4, occur 40kg of toxic waste T_1, so for one unit from M_4 occur 0,1kg of toxic waste T_1, and for x_4 units from M_4 occur $0,4x_4$ kg of toxic waste T_1. Summing up the quantities of toxic waste and taking into account the fact that the current legislation limits the amount of toxic waste T_1 at 21kg, we get the first restriction attached to the problem to be solved, namely:

$$0,2x_1+0,2x_2+0,3x_3+0,4x_4 \leq 21 \qquad\qquad (2)$$

By the same reasoning, looking every time to the maximum limits of the toxic waste T_1, T_2, T_3 respectively of 21kg, 6kg şi 14kg, we obtain all the restrictions in which the case study has to be solved.

Thus, ***the restrictions of the problem***, imposed by legislation that limits the amount of toxic waste T_1, T_2, T_3 are:

$$0,2x_1+0,2x_2+0,3x_3+0,4x_4 \leq 21;$$
$$0,1x_1+0,3x_2+0,2x_3 \leq 6;$$
$$0,2x_1+0,4x_2+0,3x_3+0,1x_4 \leq 14;$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0.$$

Mathematical, solving the problem involves using the simplex algorithm (Gorski, H., 2008, pp.70-111). Problem data is entered in a simplex table:

Table no. 1

	x_1	x_2	x_3	x_4	
1	,2	,2	,3	,4	1
2	,1	,3	,2		
3	,2	,4	,3	,1	4
	15	60	4	20	

We choose from the interior of the table a nonzero element, which will be called pivotal element, with which is performed a modified Gauss-Jordan step, thereby achieving a new table. We follow the next rules that compose the modified Gauss-Jordan step:

R1. Number of line changes with the number of column (for example, if the number 0,2 from the first line is chosen as pivot, the first column, then in the next table y_1 goes in the place of x_1 above the table, and x_1 in the place of y_1 on the left of the table).

R2. Pivotal element is reversed.

R3. The elements from the line with the pivotal element are divided to the pivot.

R4. The elements from the column with the pivotal element change their sign and are divided to the pivot.

R5. All other elements are calculated by the rule of the second order determinant, taking as principal diagonal the one that contains the pivot.

Example. If we choose as pivot element 0,2 from the first line, first column, then the element 0,3 from the line 2 column 2 will be replaced by

$$\frac{\begin{vmatrix} 0,2 & 0,2 \\ 0,1 & 0,3 \end{vmatrix}}{0,2} = 0,2.$$

We repeat removal of independent variables x_i choosing another pivot but not from the line and column where it has been previously selected.

Next, for finding the solution of the problem, to positive the column of the free terms, by choosing positive pivot elements from these lines and negativation of the line of f by choosing positive pivot elements from the lines where are the coefficients of f.

In the last table feasible will be found in the last line and last column the optimum of the objective function. But, the working time in the case of the direct calculus, on paper, can be reduced if there is used software that can solve linear programming problems, software as WinQSB, LINGO etc.

In the followings, we will present the solving of the problem, of the case study, by using the WinQSB software, software which is based on this algorithm.

3. Solving the case study using winqsb

We introduce the data of the problem, of the case study, under matrix form in the module *Linear and Integer Programming* of WinQSB software, after we first specified number of variables and constraints, respectively 4 and 3, criterion for the objective function, of maximization and the type of the variables, respectively integers, in the dialog box opened by selecting option *File →New* from the specified module (Figure no. 1):
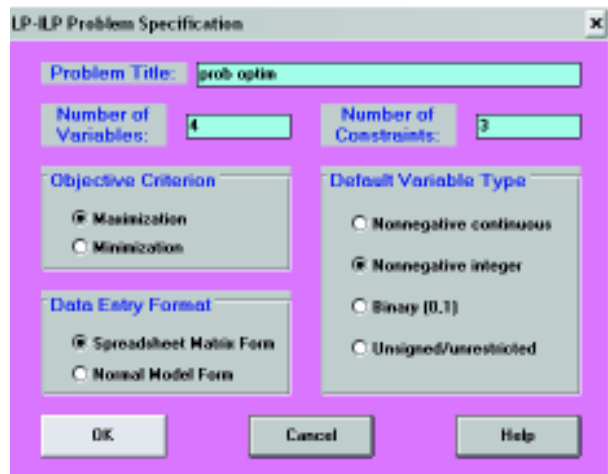
Figure no. 1. Definition of the problem

It will open a window where you can enter the objective function coefficients and the ones of the restrictions of the problem that is to bring the problem to be solved in matrix form or normal (Figure no. 2):

Variable -->	X1	X2	X3	X4	Direction	R. H. S.
Maximize	15	60	4	20		
C1	0.2	0.2	0.1	0.4	<=	21
C2	0.1	0.3	0.2	0	<=	6
C3	0.2	0.4	0.3	0.1	<=	14
LowerBound	0	0	0	0		
UpperBound	M	M	M	M		
VariableType	Integer	Integer	Integer	Integer		

Figure no. 2. Introducing the coefficients of the objective function and the restrictions

From menu *Solve and Analyse* we select and access the command *Solve the Problem* or *Solve and Display Steps* or *Graphic Method* for solving the problem, the case study, displaying the result and the sensitivity analysis (Figure no. 3):

16:52:55		Monday	May	02	2011
Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1 X1	0	15,0000	0	-5,0000	at bound
2 X2	20,0000	60,0000	1.200,0000	0	basic
3 X3	0	4,0000	0	-36,0000	at bound
4 X4	42,0000	20,0000	840,0000	0	basic
Objective	Function	(Max.) =	2.040,0000		
	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
Constraint					
1 C1	20,8000	<=	21,0000	0,2000	0
2 C2	6,0000	<=	6,0000	0	200,0000
3 C3	12,2000	<=	14,0000	1,8000	0

Figure no. 3. Solution of the problem and the sensitivity analysis

We note that the maximum of the objective function, respectively of the profit, displayed by the module *Linear and Integer Programming* is to be read from the fourth column of the table from Figure no. 3 and represents **value of 2040,00**. This value is the product of the variables from the second column and the coefficients from the third column displayed by the module *Linear and Integer Programming*. The WinQSB also offers a clear sensitivity analysis for modifying the coefficients of the objective function, and for modifying the values from the right side of the constraints (Figure no. 3, columns 4,5). The sensitivity analysis shows the limits between which the values of the coefficients of the objective function may vary and the terms from the right of the constraints- this without modifying the optimal solution, even though the form of the objective function will modify.

On the other side, charts, realised in 2D or 3D, under form of bars, circular diagrams or polygonal lines, emphasize more expressive the maximum of the objective function, and also the contribution of each variable to its achievement (Figure no. 4):
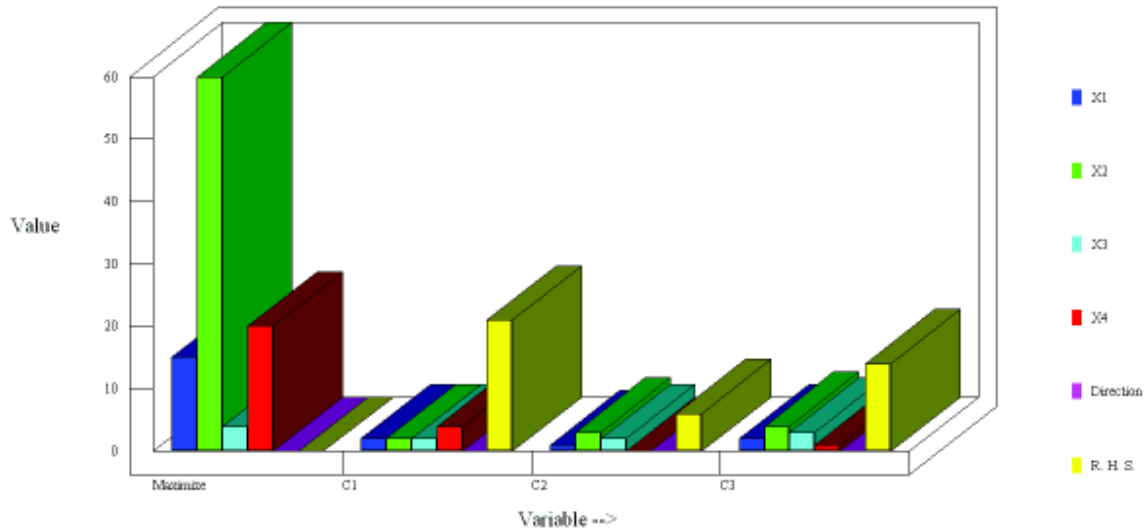
Figure no. 4. Graphical representation of the solution of the problem

4. Conclusion

The solution of the problem, displayed in the second column of the table from fig.3, is $x_1=0$, $x_2=20$, $x_3=0$, $x_4=0$. So managers, complying with the environmental legislation, should order the achievement of 20kg from medicine M_2 and 40 kg from medicine M_4, and the profit will be of 2040,00 m.u..

The linear programming method can easy be used in an analogous way to the case study presented in this article, in solving any other case study that refers to the optimization of the decisions regarding the realisation of the product plans, with respect to the conditions imposed by the sustainable development of any organisation. It has the advantage that it can quickly be understood and applied by any specialist being him engineer, economist, manager, not only by mathematicians.

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